**Homework 2- Wave Optics. Solutions**

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Deadline to give me back the homework: 23th May 2022

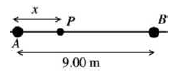
Student name (Pinyin):

Student name (Chinese):

Student number:

**Ex. 1**

As shown on the figure, a radio transmitting station operating at a frequency of 120 MHz has two identical antennas that radiate in phase. Antenna B is 9.00 m to the right of antenna A. Consider point between the antennas and along the line connecting them, a horizontal distance to the right of antenna A. a) Describe the optical path difference between both waves at P from A and B and the path difference in respect to (the refractive index of the medium of propagation is ) b) Give the condition of constructive interferences between both waves at P about the optical path difference and about the path difference between them (**it could be any refractive index , the approximation is not asked here**). c) Calculate the wavelength of the waves if they propagate in air , which is same medium for both waves ( unit: m). d)For what values of will constructive interference occur at point P (the point P considered in only between A and B, don’t consider interferences outside this range), both waves propagate in air ?



Solution:

a)

The optical path difference between the waves at P is :

Where and

And thus,

And the path difference between both waves is:

b) **Condition about the optical path difference :**

The phase difference between both waves at P is (here the phase at both point sources is the same):

Where is the wavelength in vacuum.

There are constructives interferences at P if:

where

**Condition about the path difference .**

There are constructive interferences at P if:

where

Another way more faster, is simply to use the condition about the optical path difference:

c)

The wavelength of both waves in a medium of refractive index is:

In air, the velocity of propagation of the electromagnetic wave is:

d)

We must to find the condition of constructives inteferences about :

For ,

For ,

For ,

For ,

For ,

For ,

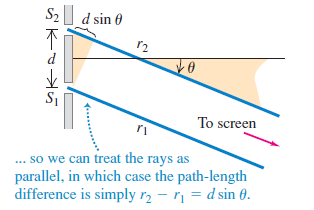
or ,

There is no other constructive interferences to consider, because the point P will be outside the range asked (here between A and B).

**Ex. 2**

Two thin parallel slits that are 0.0116 mm apart are illuminated by a laser beam of wavelength 585 nm. At and the phase is the same for both waves (a) On a very large distant screen (as shown on the figure the deviation angle is the same for both rays), what is the number of bright fringes (those indicating complete constructive interference), including the central fringe and those on both sides of it? Solve this problem without calculating all the angles! (*Hint:* What is the largest that can be ?) (b) At what angle, relative to the original direction of the beam, will the fringe that is most distant from the central bright fringe occur?

**Warning:** There are constructive interferences between waves from both slits if where ,… and is the wavelength in the medium of propagation. You don’t have to demonstrate this result. The screen is so far from the slits that two interfering waves at the screen have same angle , as shown on the figure.



**Solution:**

1) The condition of constructive interferences between all the waves with same deviation angle for is where

For , we have:

The largest on a screen is 19. They are bright fringes.

b) the most distant fringe has .

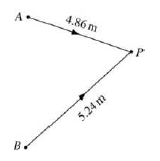
The largest angular deviation is:

Take care that for large angles, it is not possible to apply the approximation .

**Ex. 3.**

As shown on the figure, coherent sources and emit electromagnetic waves with wavelength 2.00 cm. Point P is 4.86 m from source A and 5.24 m from source B. The medium of propagation is the same for both waves (and can have any refractive index) a) Describe the phase difference between the waves emitted by both sources at P in respect with the difference **of path travelled** between them and their wavelength (the phase of the waves at A and B is the same) 2) Calculate the phase difference at between these two waves at P. 3) If both electromagnetic waves at P don’t have the same wavelength, what can you conclude?

Warning: take care to not confuse path travelled and the optical path, the wavelength in any medium of propagation and in vacuum.



**Solution:**

1)The phase difference between the waves at P is :

where is the wavelength in vacuum, is the wavelength in the medium of propagation, is the optical path difference between the waves at P.

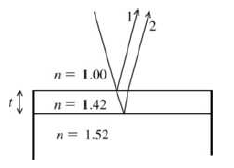
2)

3)

If both waves don’t have the same wavelength, they don’t interfere together but they overlap at P, which means there could be superposition of the waves.

**Ex. 4**

We want to find what is the thinnest film of a coating with on Glass () for which destructive interference of the red component (650 nm in vacuum and air) of an incident white light beam in air can take place by reflection. a) Describe the difference of path difference between the two waves 1 and 2 shown on the figure (**we consider normal incidence, the angle of incidence of the surface of the film is zero,** unlike what is shown on the figure to distinguish more easily both rays). Then, give a condition about the thickness of the film to have destructive interference between both waves, in terms of the wavelength and an interference order . Explain why we don’t consider interference order b) What is the thickness of the thinnest film to observe the interferences for red component of the incident light ?



**Solution.**

a)Both rays have a half-wave loss due to the reflection (because and ) and we consider the case of normal incidence, which means the path difference between them is :

The phase difference between both waves is:

Where is the wavelength in the medium of propagation (here the medium with refractive index )

There are destructive interferences between both rays if:

where We can’t consider the cases because the thickness and the wavelength are positive numbers.

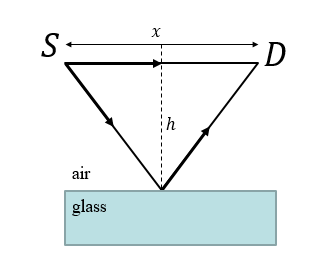
b)The thinnest film to observe destructive interference between both waves has the thickness corresponding to :

Using and , we obtain:

**Ex. 5.**

As shown in the figure, a source of monochromatic light and a detector are both located in air a distance above a horizontal plane sheet of glass and are separated by a horizontal distance . Waves reaching directly from interfere with waves that reflect off the glass. The distance is small compared to so that the reflection is at close to normal incidence. (a) Show that the condition for constructive interference is and the condition for destructive interference is , where (Hint: Take into account the half-wave loss on reflection.) (b) Let and .What is the longest wavelength for which there will be constructive interference?

About the refractive index of air, use .



**Solution:**

a)

The phase difference between both waves at the detector D is:

where is the wavelength in vacuum and is the optical path difference between the waves, from S, where they have the same phase to D:

where is the path difference between the waves at D and is the extra-path due to the reflection (half-wave loss), is the path difference between both waves at D considering the extra-path due to the reflection.

There are constructive interferences between both waves at D if , i.e. if

There are destructive interferences between both waves at D if , i.e. if

The distance from the impact at the glass to D is: .

The path difference between both waves is (if we don’t consider the extra-path due to the reflection):

If we consider the half-wave loss at the reflection, the path difference between both waves is:

There are constructive interferences if:

Where

It is not possible here to have a negative value of m here because:

And of course .

There are destructive interferences if:

Where

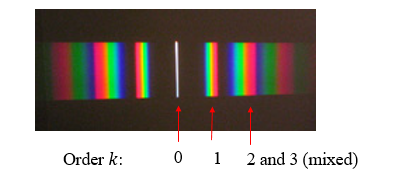
b)

There are constructive interferences if:

The longest wavelength is for :

**Ex. 6. Diffraction grating and white light.**

As shown on the figure, a white light pass through a diffraction grating with 600 lines/mm. The spectrum is observed at a screen. a) Calculate the grating spacing between two adjacent lines (unit: m) (i.e. the distance between the center of two adjacent lines). b) Describe the condition of constructive interferences between all the waves with same deviation angle at a point of the screen for any order k () about (you don’t have to demonstrate the result obtained in lecture). c) For interference order , find the angular deviation (unit: radians and degree) for 400 nm violet light and 700 nm red light (unit: rad, deg), using the paraxial approximation . d) Describe the maximum order which can be seen on the screen in terms of and the wavelength (the light propagates through air, , the wavelength in vacuum). **The figure is not to be considered** (this maximum order could be greater than 3). d) Describe the number of bright fringes corresponding to one wavelength which will be seen on the screen in terms of



**Solution**

a)The spacing grating is:

b)

Where

The condition about constructive interferences between all the waves of same deviation angle is:

c)

We consider the paraxial approximation: , we obtain:

Take care that in this equation, is expressed in radians.

We consider the order , we obtain:

For (violet light):

For (red light):

d)

where

The maximum order is such as:

Take care that the order is an integer, if you have to calculate it.

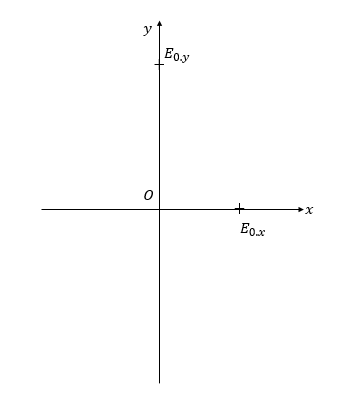
e) The number of bright fringes corresponding to one wavelength is:

We obtain:

Take care that a number of fringes is an integer, if you have to calculate it.

**Ex. 7. *A particular polarization of light.*** We consider a plane wave propagating in the +z-direction. The x-component and y-component of the electric field are described as follows: and ( because of the direction of propagation). As shown in both equations, the electric field amplitude is the same for both components: . The phase difference between both components is . a) Describe the magnitude of the electric field (demonstrate it, don’t use directly a result seen in classroom). b) The electric field for this polarization can be described as follows: , where are the unit vector along the +x-direction and the +y-direction. What is the function . c) Which kind of polarization is described in this exercise ? d) Now, we consider that the amplitude of both components is not the same: and with . Draw on the graph with x-axis and y-axis the change of the electric field when the wave propagates. The amplitudes and shown on the figure must be used (so here ). What you can say about the shape described by the electric field ?

**About the question d),** it could help you to use a scientific calculator but please to try to do the question without it. The origin of space and time is chosen such as , draw the electric field at , think about the electric field at particular times, such as ,, etc.



**Solution**

a)

The electric field magnitude is:

The direction of propagation is the +z-direction, so the electric field can’t have a z-component: .

The x- and y- components of the electric field are:

The y-component of the electric field can be described by:

I remind you that and

The electric field magnitude is then:

The case of a phase shift between both components of and amplitude of both components are the same is a particular case where the electric field magnitude is constant when the wave propagates. The electric field vector describes a circle, i.e. the polarization is circular.

b)

And

Thus, the function describe the phase:

c)

The electric field has a circular polarization.

d)

The electric field components are now described by:

where

The origin of space and time is chosen such as, and we want to describe the electric field at :

Take care that the electric field don’t describe a circle, here!

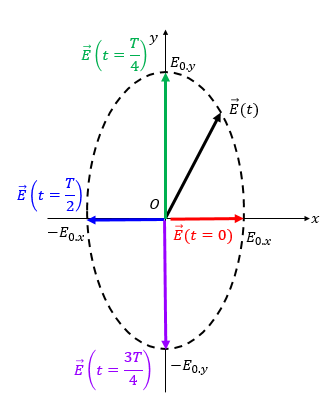
At time ,

At time such as , i.e. at time where is the period of the wave,

At time such as , i.e. at time ,

At time such as , i.e. at time ,

At time such as , i.e. , the electric field is the same than at time .



Here is the electric field describes an ellipse which the axes are the x-axis and the y-axis (this particular case of ellipse is because the phase shift between components is , here).